International Discovery of Ali π

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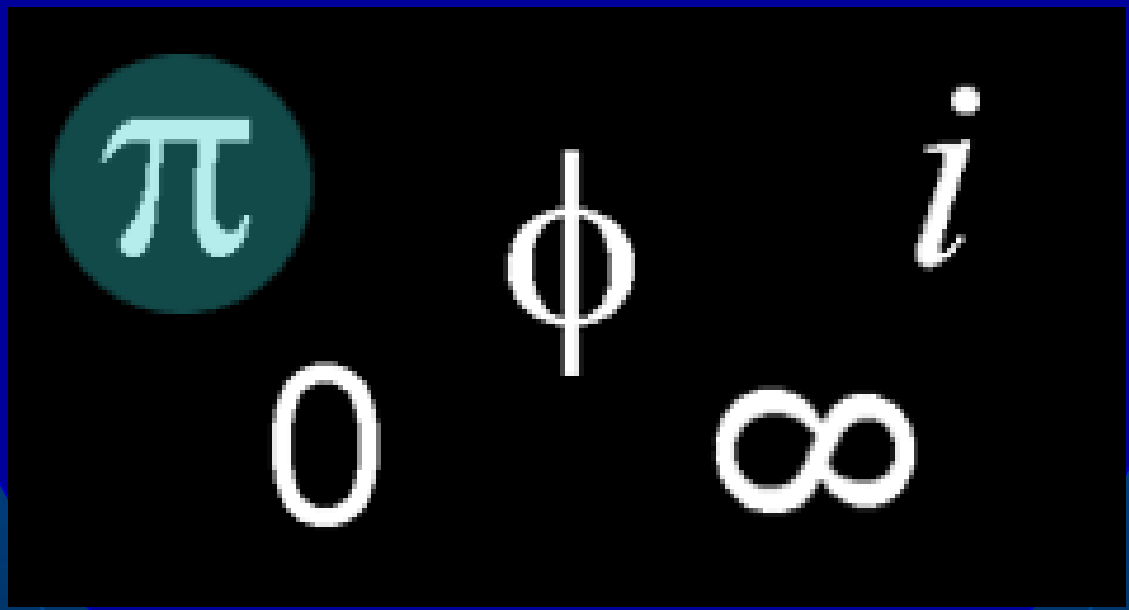
Phase 1

Introduction and History of Most Famous Mathematical Perfect Constant

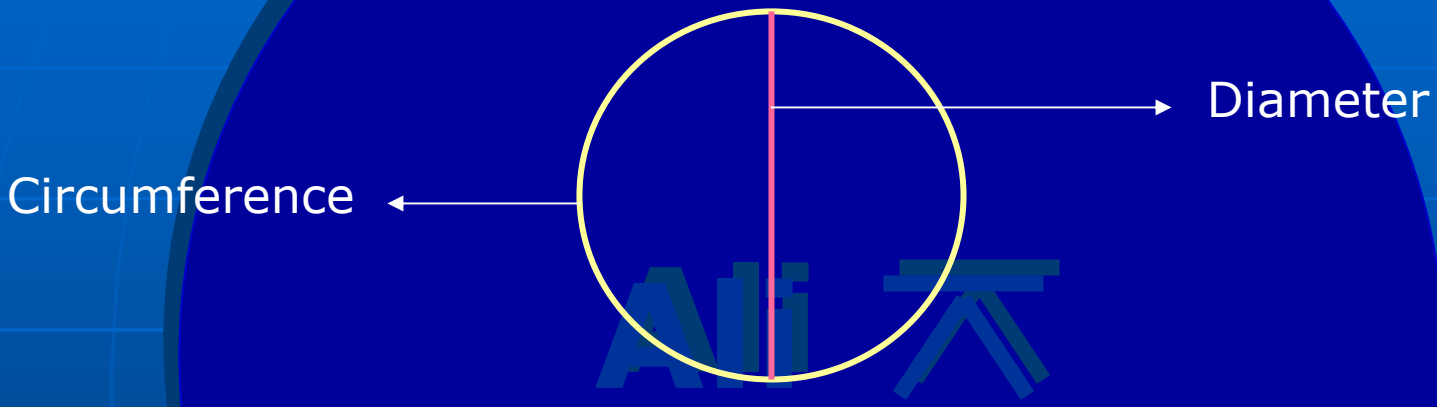
5 Important and Famous Mathematical constants



Most Famous Mathematical Constant

 - Most Famous Ratio in Mathematics and in the History of Mankind

Universal Constant Ratio



π **= Circumference of a circle/Diameter of a circle**

Circle – Mathematical Definition

A circle is a simple closed curve that divides the plane into an interior and exterior. It has a perimeter, called a circumference of length $2\pi r$ and encloses an area of $2\pi r^2$. The set of all Points in a plane, at a given distance, called the radius, from a fixed point, called the center.

"Circle" comes from the Latin – 'Circus', which refers to a large round or rounded oblong enclosure in which the famous Roman chariot races were held.

Perfect Circle and π

"The circle is one of the noblest representation of the Deity, in his noble works of human nature. It bounds, determines, governs, and dictates space, bounds latitude and longitude, refers to the sun, moon, and all the planets, in direction, brings to the mind thoughts of eternity, and concentrates the mind to imagine for itself the distance and space it comprehends. It rectifies all boundaries; it is the key to information of the knowledge of God."

John Davis – The Measure of the Circle, 1854

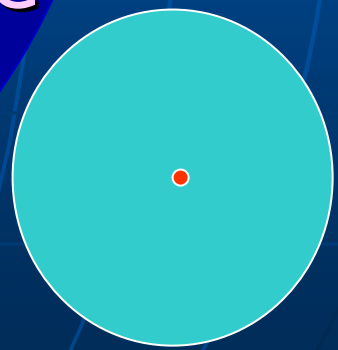
Mysterious π

"Probably no symbol in mathematics has evoked as much mystery, romanticism, misconception and human interest as the number pi."

William L. Schaaf
(Nature and History of Pi)

Center of a Circle

In geometry, the center of a circle is the point equidistant from the points on the edge. Similarly the center of a sphere is the point equidistant from the points on the surface and the center of a line segment is the midpoint of the two ends



Radius of a circle

- In classical geometry, a radius of a circle or sphere is any line segment from its center to its boundary. The radius of a circle or sphere is the length of any such segment. The radius is half the diameter.
- The relationship between the radius and the circumference of a circle is:

$$r = C/(2 \times \pi)$$

Where **r = radius of a circle**

Pi = Circumference divided by diameter of a circle

C = circumference of a circle.

Diameter of a circle

- In geometry, a diameter of a circle is any straight line segment that passes through the centre and whose endpoints are on the circular boundary. The diameter is the length of such a line segment.

- All the diameters of a given circle have the same length. This length is twice the radius of a circle. The diameter of a circle is also called the longest chord that the circle has.

The diameter of a circle – d = 2 x r

where 'r' = radius of a circle

Circumference of a circle

The circumference of a circle is the distance around the circle. Circumference is a kind of perimeter.

The circumference of a circle can be calculated from its diameter using the formula:

$$C = \pi \times d$$

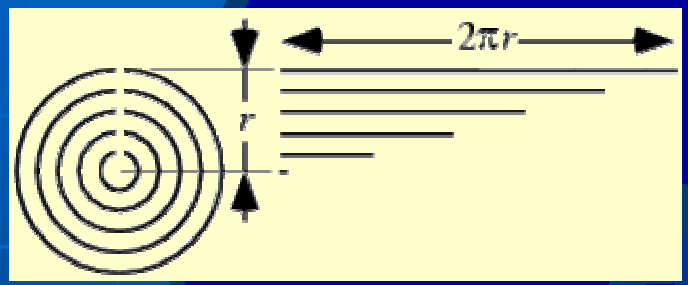
where'd'= diameter of a circle

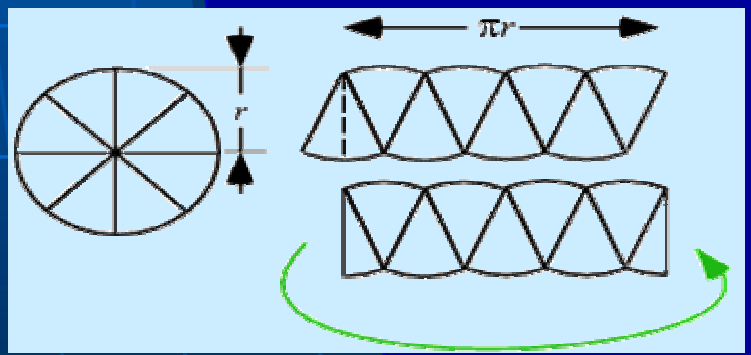
$$C = 2 \times \pi \times r$$

where 'r'= radius of a circle.

and Pi = constant ratio of the circumference and the diameter of a circle.

Circumference of a Circle





Circumference of a circle = $2\pi r$

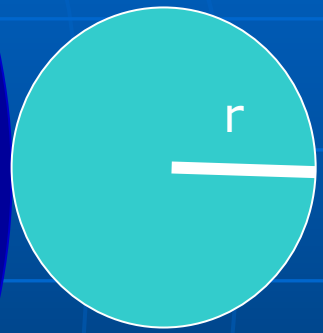
= πd

Area of a Circle

The Area of a Circle is expressed in the equation as:

Area of a Circle = π x (r x r)

= (1/4) x π x (d x d)



Where r = radius of a circle

d = diameter of a circle.

And Pi = Circumference divided by the diameter of a circle

= C/d

Chords and Circle

- Chords equidistant from the center of a circle are equal in length.
- The equal length chords are equidistant from the center.
- The perpendicular bisector of a chord passes through the center of a circle.
- If a central angle and an inscribed angle of a circle are subtended by the same chord and on the same side of the chord, then the central angle is twice the inscribed angle.

Definition of a Great circle

- **A great circle is a circle on the surface of a sphere that has the same circumference as the sphere, dividing the sphere into 2 equal hemispheres.**
- We can also define the great circle on a sphere **is a circle on the sphere's surface whose center is the same as the center of the sphere. It is the intersection of a sphere with a plane going through its center.**
- **A Great Circle is the 'largest Circle' that can be drawn on a given sphere. The great circle on the spherical surface is the path with the smallest curvature and therefore an arc – an orthodrome is the shortest path between 2 points on the surface and the distance between any two points on a sphere is known as the great – circle distance.**

Compass and Circle

- **Early science, particularly geometry and astronomy/ astrology, was connected to the divine for most medieval scholars.**

- **- The compass in this 13th Century manuscript is a symbol of God's act of Creation, as many believed that there was something intrinsically "divine" or "perfect" that could be found in circles.**

π - Different Names in History

1. Ludophian Number

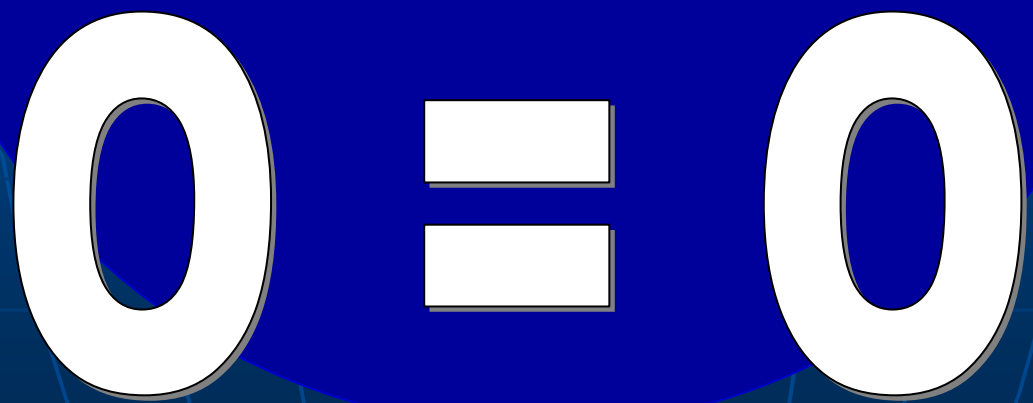
2. Archimedes Constant

3. Greek Pi - π

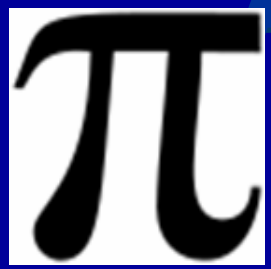
Zero is a Circle

- Zero is regarded as the biggest discovery in Mathematics and sciences.
- Zero's shape is Circle.

O (Zero) = O (Circle)



Zero / One



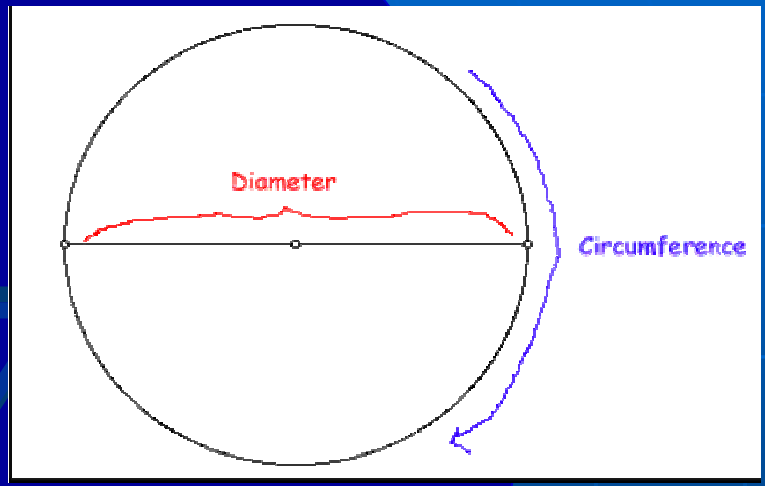
= 0 / 1

Geometry and Numbers

- **Geometry:**
  It is the study of the properties of shapes and the space around them, from a simple triangle to the most complex solid. For example, the study of shapes like triangle and ico-sahedron and the relationship between them.

- **Numbers:**
  A number is a mathematical concept used to describe and assess quantity. It is an abstract entity representing a quantity, used to express how many things are being referred to, or how much there is of some thing or property; an arithmetical value corresponding to a particular quantity of something.

Significance of Pi



Historians estimate that by 2000 B.C. humans had noticed that the ratio of circumference to diameter was the same for all circles. This discovery hinged on the idea of proportion - in this case humans noticed that if you double the distance "across" a circle, then you double the distance "around" it. In today's algebraic notation this implied the formula

Significance of Pi (Cont..)

The significance of this discovery is clear:

- **Circles are everywhere - in the sun, the moon, the pupils of our eyes, the most basic religious rituals and the earliest man-made structures.**
- Achieving a greater mathematical understanding of Pi would lead to scientific and technological advances that would further the development of civilization, as well as creating some very interesting problems in pure mathematics

First Value of Pi calculated in Egypt – 3.16.....

- First Value of π calculated by **Egyptian scribe Ahmes**

  = 256/81
  = 3.160.....
  = **3.16.....**

  First ever value found around 1650 BC.
- Ahmes began scroll with the words:

"The Entrance into the Knowledge of All Existing Things"

and remarks in passing that he composed the scroll 'in likeliness to writings made of old.' Towards the end of the scroll, which is composed of various mathematical problems and their solutions, the area of a circle is found using a rough sort of pi."

First Value of Pi calculated in Egypt – 3.16.....

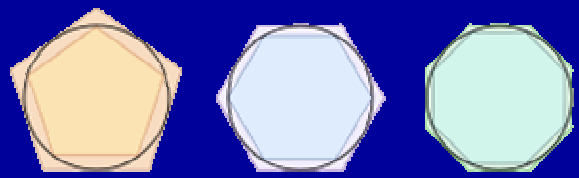
- First Value of π calculated by Egyptian scribe Ahmes

= 256/81
= 3.160493827.....
= 3.16.....

First ever value found around 1650 BC.

A portion of the Rhind Papyrus

Archimedes‘ Constant



Principle of Archimedes' method to approximate π

- Around 200 BCE, Archimedes of Syracuse, one of the greatest mathematicians of the ancient world, approximated that pi is somewhere about 3.14 in fractions because Greeks did not have decimals.
- Archimedes was the first to give a scientific method for calculating **pi** to arbitrary accuracy.
- Archimedes of Syracuse discovered, by considering the perimeters of 96-sided polygons inscribing a circle. When a circle's diameter is 1, its circumference is **pi**
- Archimedes knew that he had not found the value of **pi** but only an approximation within those limits.

Famous Mathematicians Who Tried to Calculate Pi in the History

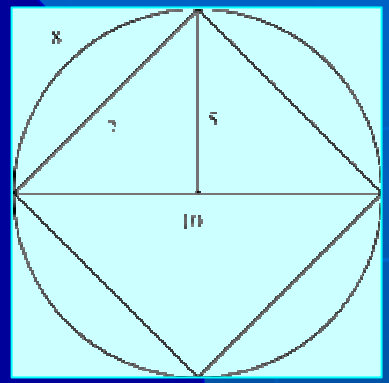
1. Egyptian Rhind Ahmes
2. Babylonians
3. Archimedes
4. Hon Han Shu
5. Brahmagupta
6. Al – Khwarizmi
7. Fibonacci -"Pi from now on calculated in decimal places."
8. Madhava of Sangamagrama – discovered the infinite power series expansion of pi
9. Jamshid Masud Al Kashi
10. Ludolph Van Ceulen

Famous Mathematicians Who Tried to Calculate Pi in the History (cont..)

11. Isaac Newton
12. Leonhard Euler – used letter pi in his book and pointed that pi may be transcendental
13. Johann Heinrich Lambert – who proved that pi is irrational
14. Rutherford
15. Srinivasa Ramanujan
16. D.F. Ferguson
17. Felton
18. J.Guillord
19. Yasumasa Kanada – 1.24 trillion places on Hitachi SR8000/MPP (64 nodes), 600 hours

Pi = 3.14159265358979323846264338327950288419716939937510...1.24 trillion decimal places

USA – Indiana Pi Bill – 1897 AD

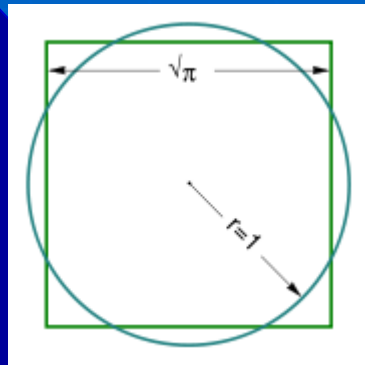


- USA - Indiana Pi Bill – 1897 AD

- **Goodwin's model circle as described in section 2 of the bill. It has a diameter of 10 and a circumference of 32; the chord of 90° has length 7**
- The **'Indiana Pi Bill' of 1897 AD , which never passed out of committee, has been claimed to imply a number of different values for Pi, although the closest it comes to explicitly asserting one is the wording, ' the ratio of the diameter and circumference is as five-fourths to four' which would make Pi = 3.2**

Pi = 3.2 (as proposed by Indiana Pi Bill in 1897 AD in USA).

Squaring the circle and circle squarers

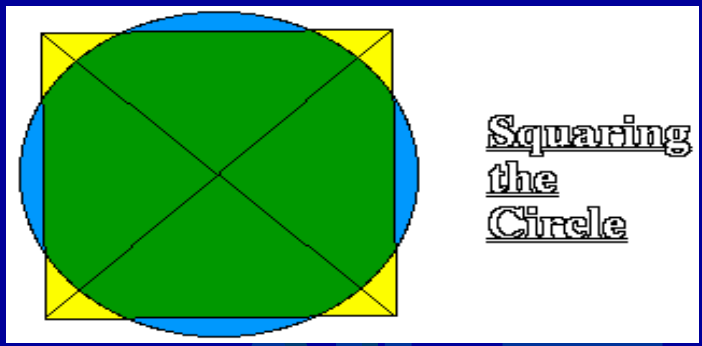


Squaring the circle: the areas of this square and this circle are equal

The first mathematician, who is on record as having attempted to square the circle is ***Anaxagoras Plutarch,*** in his work On Exile which was written in the first century AD, says:

"There is no place that can take away the happiness of a man, nor yet his virtue or wisdom."

Squaring the Circle



- Squaring the circle **means to make the square** with the exact and equal area of a circle. As we cannot calculate the area of a circle exactly with the irrational and transcendental value of Pi, so squaring the circle became impossible solution till now.

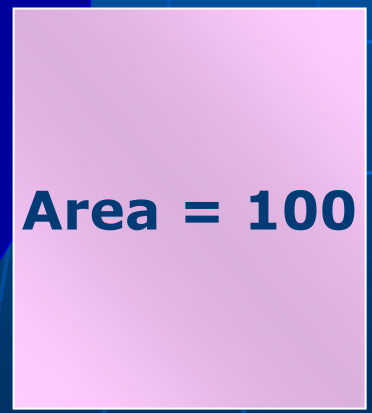
- Squaring the Circle is **declared impossible** with the proof that is a transcendental number – Dream of 4000 years to square the circle was put to death.

'God exists since mathematics is consistent, and the devil exists since we cannot prove it.'

 (Andre' Weil, French Mathematician)

Early Circle Squarers

- From that time, the expression, '**circle-squarers**' came into usage. Indeed we know of the work of a number of mathematicians on this problem during that period:
- **Oenopides**
- **Antiphon**
- **Bryson**
- **Hippocrates**
- **Hippias**



Ahmes Papyrus – Problem – 50 Squaring the Circle

- The problem – 50 reads as:

'A circular field has diameter 9 khet. What is its area?'

- **Ahmes' solution is:**

'Take away thou 1/9 of it, namely 1; the remainder is 8. Make thou the multiplication 8 times 8; becomes it 64; the amount of it, this is, in area 64 setat.'

- When we take this Ahmes solution as a general formula, then in modern notation, we get the formula for the area A of a circle of a diameter d as:

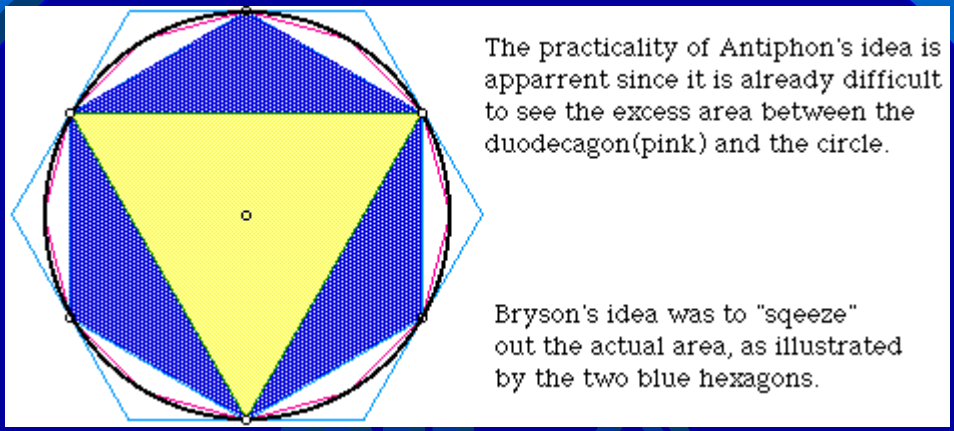
$$A = \text{square of } (d - d/9) = (64/81)\,(d)^2$$

Aristopenes and Squaring the Circle

- **There is a reference in a play *Birds,* written by Aristopenes in about 414 B.C.**

"Meton responded to Peisthetaerus: Well I now apply the straight rod – so – thus squaring the circle: and there you are. In the center you have your market place: straight streets leading into it, from here, from here, from here. Very much the same principle, really, as the rays of a star: the star itself is circular, but sends out straight rays in every direction."

Antiphon and Bryson Squaring the Circle



The practicality of Antiphon's idea is apparrent since it is already difficult to see the excess area between the duodecagon(pink) and the circle.

Bryson's idea was to "sqeeze" out the actual area, as illustrated by the two blue hexagons.

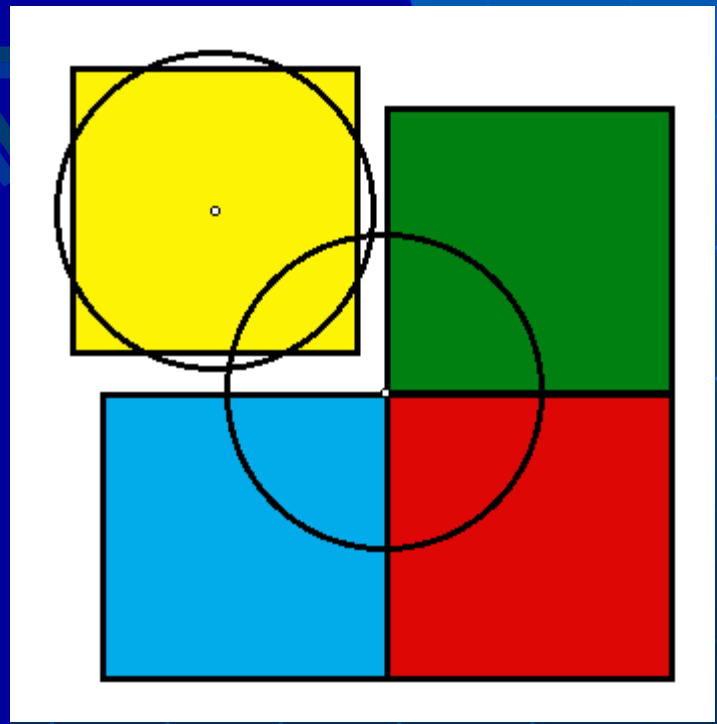
Antiphon and Bryson both produced arguments relating to squaring the circle which were to prove important in the future development of mathematics. Bryson, a student of Socrates, took the obvious approach of using inscribed and circumscribed polygons to squeeze the area of the circle. Imagine drawing a square inside a circle such that all four of its corners touch the circle, and a second square around the circle such that each side is tangent to the circle. The area of the circle would be somewhere between the areas of these two squares. By using hexagons, pentagons, and so on, the circle's area is constrained to smaller and smaller ranges. Bryson tried hard but failed to prove that it was possible to compute the circle's area exactly by taking this approach far enough.

Squaring the circle – as a metaphor

Squaring the circle has become a metaphor in the history of mankind. For example, in Spanish, the expression, **"descubriste la cucdratura del circulo"** meaning,

'You discovered the quadrature of the circle'

is often used to dismiss claims that someone has found a simple solution to a particularly hard or intractable problem.



Were the ancient Greeks idiots?

John Ruch writes in his article in 2004 about the squaring of the circle as:

"It refers to the geometrical attempt -- now known to be impossible – to create a square that has the exact same area as a given circle. But what the common phrase refers to is an ancient Greek theoretical formulation. It's actually one of three conundrums from ancient Greek geometry, all highlighting different theoretical difficulties. Not surprisingly, it's also the one with the catchiest name and involving the simplest shapes. The other two are doubling or duplicating the cube and trisecting an angle.

Squaring the circle is also the 'most impossible' of the three – the other two involve operations that are possible in certain specialized circumstances. Now, the fact is, many Greek mathematicians explored other ways of squaring the circle, using marked points, dynamic curves, conical sections and other geometrical arcane. Such methods can get you pretty close, whereas using the arbitrary straightedge and compass method gets you nowhere. So why the drawing rules?

Were the ancient Greeks idiots? (Cont..)

- As a simplified thought experiment, it also emphasizes the key difficulty of the whole problem: defining the number Pi, which is really what the whole squaring the circle stunt is all about. Pi is the ratio between the circumference of any circle and its diameter. Pi is also, therefore, a key number in determining the area of a circle- area equals Pi times the radius squared.

- Problem is, Pi is not a whole number ratio. It's an endlessly repeating decimal monstrosity that begins 3.1415…… and continues on without a pattern, apparently forever. It has been calculated to many billions of decimal places. In short, a circle – a defined, geometric figure – contains a mathematical number that is literally infinitely impossible. It is a grand mystery, and it means that we can't even say exactly what the area of a circle is, let alone what a square of the same area would look like.

Were the ancient Greeks idiots? (Cont..)

- **Put in math lingo, pi and its square root can't be expressed by any finite set of mathematical operations. That includes any series of geometrical straight lines, like a square.**

- **Put in plain English, what squaring the circle really means is drawing a line that is the square root of Pi in length, to use as the sides of the square. But the square root of Pi is also an endlessly repeating decimal. You can't draw a finite square with infinitely long sides, now, can you?**

- **Were the ancient Greeks idiots? Did they really think it would be fun to try to draw a finite line representing an infinite ratio? They were a bunch of lazy slaveholders with plenty of time to sit around and think up weird things, so knowing better probably wouldn't have stopped them – but they didn't know better. Squaring the circle was an attempt to trap Pi and squeeze it for information."**

The circle is one of the greatest Enigmas of Mathematics

- **It is defined as the set of points in a given plane at a given distance from a center point.**
- Practically, **a compass** is an excellent tool for describing such a circle. It is one of the simplest concepts, a cornerstone in the edifice of mathematics. Yet, it eludes mathematical exactness up till now. It is not difficult to see that why **so many wise men** pondered the problem in hopes of imposing order upon an irrational Pi and squaring the circle with a compass and a straight edge.
- **Squaring the circle** is one of the **three great problems of Classical Geometry**, along with the trisection of the angle and the duplication of the cube.

Enigma Of Mathematics

Irrational Value of Pi

3.14159265358979323846264338327950288419716939937510582
97494459230781640628620899862803482534211706798214808 6
51328230664709384609550582231725359408128481117450284 1
02701938521105559644622948954930381964428810975665933 4
46128475648233786783165271201909145685669234603486104 5
43266482133936072602491412737245870066063155881748815 2
09209628292540917153643678925903600113305305488204665 2
18414695194151160943305727036575959195309218611738193 2
61179310511854807446237996274956735188575272489122793 8
18301194912983367336440656643086021394946395224737190 7
02179860943702770539217176293176752384674818467669405 1
32000568127145263560827785771343751957781857780532171 2
26806613001927876611195909216420198 ……

Irrational Pi = 3.1415……. 1.24 Trillion Decimal places till 2002 AD

Once Goethe quoted:

"It has been said that figures rule the world. Maybe, but I am sure that figures show us whether it is being ruled well or badly."

Irrational Pi on Irrational Path

Irrational and Transcendental - 1.24 trillion decimal places

By 2002 AD, **Yasumasa Kanada** and team, Hitachi SR8000 / MPP (64 nodes), 600 hours found the value of Pi up to:
1,241,100,000,000 decimal places of Pi.

"Exploring Pi is like exploring the universe."

--David Chudnovsky

Un Ending Irrational Pi

Irrational Pi –

3.14159265358979323846264338327950288419 71…1.24 trillion decimal places

"Knowledge is a Point,
Ignorant people spread it."
(Imam ALI)

Irrational and Transcendental

- **Johann Heinrich Lambert proved that pi is an irrational number and it is a lie to look for rational pi in 1761 because it cannot be written as the ratio of two integers.**

- **In 1882, Ferdinand von Lindemann showed that pi is also transcendental number, which means that there is no polynomial with rational coefficients of which Pi is a root.**

Mathematicians in the search of π

Many formulas in analysis contain pi, including infinite series and infinite product representations, integrals, and so-called special functions.

- **The area of the unit disc:**

$$2\int_{-1}^{1}\sqrt{1-x^2}\,dx=\pi$$

- **Half the circumference of the unit circle**

$$\int_{-1}^{1}\frac{dx}{\sqrt{1-x^2}}=\pi$$

Mathematicians in the search of π (Cont…)

- **François Viète, 1593 (proof)**

$$\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{2+\sqrt{2}}}{2}\cdot\frac{\sqrt{2+\sqrt{2+\sqrt{2}}}}{2}\cdots=\frac{2}{\pi}$$

- **Leibniz' formula (proof):**

$$\sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1}=\frac{1}{1}-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\frac{1}{9}-\cdots=\frac{\pi}{4}$$

- **Wallis product, 1655 (proof)**

$$\prod_{n=1}^{\infty}\left(\frac{n+1}{n}\right)^{(-1)^{n-1}}=\frac{2}{1}\cdot\frac{2}{3}\cdot\frac{4}{3}\cdot\frac{4}{5}\cdot\frac{6}{5}\cdot\frac{6}{7}\cdot\frac{8}{7}\cdot\frac{8}{9}\cdots=\frac{\pi}{2}$$

Mathematicians in the search of π (Cont…)

- **Gamma function evaluated at 1/2**

$$\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$$

- **Sterling's approximation**

$$n! \sim \sqrt{2\pi n}\left(\frac{n}{e}\right)^n$$

- **Euler's identity (called by Richard Feynman "the most remarkable formula in mathematics")**

$$e^{i\pi} + 1 = 0$$

Efficient infinite series- Calculating value of π

$$\sum_{k=0}^{\infty} \frac{k!}{(2k+1)!!} = \frac{\pi}{2}$$

$$12\sum_{k=0}^{\infty} \frac{(-1)^k(6k)!(13591409+545140134k)}{(3k)!(k!)^3 640320^{3k+3/2}} = \frac{1}{\pi}$$

$$\frac{2\sqrt{2}}{9801}\sum_{k=0}^{\infty} \frac{(4k)!(1103+26390k)}{(k!)^4 396^{4k}} = \frac{1}{\pi}$$

The following are good for calculating arbitrary binary digits of pi:

$$\sum_{k=0}^{\infty} \frac{1}{16^k}\left(\frac{4}{8k+1} - \frac{2}{8k+4} - \frac{1}{8k+5} - \frac{1}{8k+6}\right) = \pi$$

$$\frac{1}{2^6}\sum_{n=0}^{\infty} \frac{(-1)^n}{2^{10n}}\left(-\frac{2^5}{4n+1} - \frac{1}{4n+3} + \frac{2^8}{10n+1} - \frac{2^6}{10n+3} - \frac{2^2}{10n+5} - \frac{2^2}{10n+7} + \frac{1}{10n+9}\right) = \pi$$

Mathematicians never found the the Rational and Real Value of PI till today

What is the Rational and Exact value of π?

Computation of π

"The computation of Pi is virtually the only topic from the most ancient stratum of mathematics that is still of serious interest to modern mathematical research."

Len Berggen, Jonathan Borwein and Peter Borwein – Pi- A Source Book

Important persons who computed Pi in recent years to many digits

- **Peter B. Borwein** – 10 billionth hexadecimal with BBP algorithm. A.G.M. with quartic algorithm.
- **Jonathan M. Borwein** - A.G.M. with quartic algorithm.
- **David H.Bailey** – 29 million and 10 billionth hexadecimal with all methods.
- **Fabrice Bellard** – 50 and 100 billionth hexadecimal with BBP algorithm.
- **D.V. Chudnovsky and G.V. Chudnovsky** – 1, 2 and 4 billion with Chudnovsky formula. By March 1996, more than 8 billion digits have been calculated.

Important persons who computed Pi in recent years to many digits (Cont..)

- **Guillord and Bouyer** – 250,000, 500,000, 1 million and 2 million with arctan formulas.
- **William Gosper** – 17.5 million digits with Ramanujan formula.
- **Daniel Shanks and John Wrench Jr.** – 100,265 in 1961 with arctan formulas.
- **Simon Plouffe** – 100 million hexadecimal digits with A.G.M. and 3.2 billion, 4.2 billion and 6.4 billion decimal with A.G.M. methods.
- **Yasumasa Kanada** – 1.24 trillion places on Hitachi SR8000/MPP (64 nodes), 600 hours till 2002.

Piphilology

Piphilology comprises the creation and use of mnemonic techniques to remember a span of digits of the **mathematical constant pi.**

The word is a play on Pi itself and the linguistic field of philology. Even before computers calculated Pi, memorizing a record number of digits became an obsession for some people.

The **current world record is 100,000 decimal places**, set on October 3, 2006 by Akira Haraguchi.

Role of π in our world

- **Chris Witcombe from Sweet Briar College** writes in his article on Pi as:

  "Physicists have noted the ubiquity of pi in nature. Pi is obvious in the disks of the moon and the sun. The double helix of DNA revolves around pi. Pi hides in the rainbow, and sits in the pupil of the eye, and when a raindrop falls into water pi emerges in the spreading rings. Pi can be found in waves and ripples and spectra of all kinds, and therefore pi occurs in colors and music. Pi has lately turned up in superstrings.

  Pi occurs naturally in tables of death, in what is known as a Gaussian distribution of deaths in a population; that is, when a person dies, the event 'feels pi'. It is one of the great mysteries why nature seems to know mathematics."

- **According to Havelock Ellis in his book – The Dance of Life – wrote:**

  "It is here in mathematics that the artist has the fullest scope of his imagination."

π and its Usage

- **Pi is used in Mechanical problems**, drawing, machining, etc. Pi occurs in radio signals, TV, radar, telephones, etc. Sine waves have a fundamental period of 2pi, so pi becomes vital in signal processing, spectrum analysis i.e. finding out what frequencies are in a wave you receive or send, etc.

- Everyone's favorite distribution – **normal or Gaussian** has pi in the formula and it is used in all areas of engineering to simulate unknown factors and loading conditions.

- **Pi is used in Navigation, global paths, global positioning.** When planes fly great distances they are actually flying on an arc of a circle. The path must be calculated as such in order to accurately gauge fuel use, etc. Additionally, when locating your self on a globe, pi comes into the calculation in most methods.

π and its Usage (Cont..)

- **Pi is used in Physics in cosmological constant, Heisenberg's uncertainty principle, Einstein's field equation of general relativity, Coulomb's law for the electric force, magnetic permeability of free space, etc.**

- **Pi is used in analysis including infinite series, integrals and so called special functions.** Pi is used in the area of the unit disc, half the circumference of the unit circle, Leibniz formula, Wallis product, Faster product, Symmetric formula, Bailey-Borwein Plouffe algorithm, etc

- **Pi is used in number theory** like theory of elliptic curves and complex multiplication which derives the approximation of pi, dynamical systems and ergodic theory. Pi is also used in probability and statistics.

Classical Geometry

$$C = 2\pi r,$$

where *C* is the circumference of a circle and *r* is the radius.

$$A = \pi r^2 = \frac{1}{4}\pi d^2$$

where *A* is the area of a circle and *r* is the radius

$$V = \frac{4}{3}\pi r^3,$$

where *V* is the volume of a sphere and *r* is the radius

$$A = 4\pi r^2$$

where *A* is the surface area of a sphere and *r* is the radius.

"For the things of the world cannot be made known without knowledge of mathematics." **(Roger Bacon)**

Classical Geometry(cont..)

Volume of cylinder of height *h* and radius *r*

$$V = \pi r^2 h$$

Volume of cone of height *h* and radius *r*

$$A = 2(\pi r^2) + (2\pi r)h = 2\pi r(r + h)$$

Surface area of cylinder of height *h* and radius *r*

$$V = \frac{1}{3}\pi r^2 h$$

Surface area of cone of height *h* and radius *r*

$$A = \pi r^2 + \pi r\sqrt{r^2 + h^2} = \pi r(r + \sqrt{r^2 + h^2})$$

Physics using π

- The **number Pi** appears commonly in equations describing fundamental principles of the Universe, due in no small part to its relationship to the nature of the circle and, correspondingly, spherical coordinate systems
- **The cosmological constant:**

$$\Lambda = \frac{8\pi G}{3c^2}\rho$$

- **Heisenberg's uncertainty principle**

$$\Delta x \Delta p \geq \frac{h}{4\pi}$$

Physics using π

- **Einstein's field equation of general relativity**

$$R_{ik} - \frac{g_{ik}R}{2} + \Lambda g_{ik} = \frac{8\pi G}{c^4} T_{ik}$$

- **Magnetic permeability of free space**

$$\mu_0 = 4\pi \cdot 10^{-7}\,\mathrm{N/A^2}$$

- **Coulomb's law for the electric force**

$$F = \frac{|q_1 q_2|}{4\pi\epsilon_0 r^2}$$

- **Kepler's third law constant**

$$\frac{P^2}{a^3} = \frac{(2\pi)^2}{G(M+m)}$$

Dynamical systems and Ergodic theory

- Consider the recurrence relation

$$x_{i+1} = 4x_i(1 - x_i)$$

- Then for almost every initial value $x0$ in the unit interval [0, 1]:

$$\lim_{n \to \infty} \frac{1}{n} \sum_{i=1}^{n} \sqrt{x_i} = \frac{2}{\pi}$$

- This recurrence relation is the logistic map with parameter $r = 4$, known from dynamical systems theory.

"The numbers are a catalyst that can help turn raving madmen into polite humans."

Philip J. Davis

Number theory using π

- The probability that two randomly chosen integers are co-prime is **$6/\pi^2$.**
- The probability that a randomly chosen integer is square-free is **$6/\pi^2$.**
- The average number of ways to write a positive integer as the sum of two perfect squares is **$\pi/4$.**
- In the above three statements, **"probability", "average", and "random"** are taken in a limiting sense, i.e. we consider the probability for the set of integers {1, 2, 3,…, *N*}, and then take the limit as *N* approaches infinity.
- The product of $(1 - 1/p2)$ over the primes, *p*, is **$6/\pi^2$.**
- The theory of elliptic curves and complex multiplication derives the approximation

Number theory using Pi (Cont…)

- The product of (1 − 1/*p*2) over the primes, *p*, is **6/π2.**
- The theory of elliptic curves and complex multiplication derives the approximation

$$\pi \approx \frac{\ln(640320^3 + 744)}{\sqrt{163}}$$

Which is valid to about 30 digits.

"The Science of Pure Mathematics in its modern development may claim to be the most original creation of the human spirit."

(Alfred North Whitehead)

Probability and statistics using π

- In probability and statistics, there are many distributions whose **formulæ contain *π***, including:
- Probability density function (pdf) for the normal distribution with **mean μ and standard deviation σ**

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/(2\sigma^2)}$$

- pdf for the (standard) Cauchy distribution

$$f(x) = \frac{1}{\pi(1+x^2)}$$

- Note that since $\int_{-\infty}^{\infty} f(x)\,dx = 1$ for any pdf $f(x)$, the above formulæ can be used to produce other integral formulae for π.

Probability and statistics using π (Cont...)

- A semi-interesting empirical approximation of *π* is based on **Buffon's needle problem**. Consider dropping a needle of length *L* repeatedly on a surface containing parallel lines drawn *S* units apart (with $S > L$). If the needle is dropped *n* times and *x* of those times it comes to rest crossing a line ($x > 0$), then one may approximate *π* using:

$$\pi \approx \frac{2nL}{xS}$$

- Another approximation of *π* is to throw points randomly into a quarter of a circle with radius 1 that is inscribed in a square of length 1. ***π,*** the area of a unit circle, is then approximated as

4 x (points in the quarter circle) / (total points).

Recent Famous Books and Authors on Pi

Mathematicians and scientist all over the world are still working day and night to find the real and rational value of Pi.

Some of the recent publications on Pi are:

1. **The Joy of Pi by David Blatner**
2. **Life of Pi by Yann Martel**
3. **Pi: A Biography of the World's Most Mysterious Number by Alfred S. Posamentier**

Pi: A Biography of the World's Most Mysterious Number by Alfred S. Posamentier and Ingmar Lehmann

Recent Famous Books and Authors on Pi (Cont..)

4. Pi in the sky: Counting, Thinking and Being by John D. Barrow

5. Pi by Sean Gullette – DVD

6. A History of Pi by Petr Beckman

7. Piece of Pi: Wit sharpening, Brain-bruising, Number-crunching Activities with Pi by Naila Bokhari

8. The Number Pi by Pierre Eymard

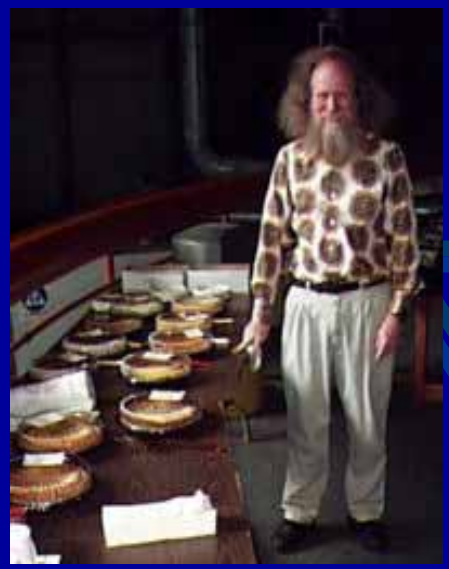
9. Pi- Unleashed by Jorg Arndt, Christoph Haenl, C. Lischka and D. Lischka

10. Pleasures of Pi, e and other interesting Numbers by Y.E.O. Adrian

11. Pi to Five Million Places by Kick Books

12. Easy as Pi? An introduction to Higher Mathematics by Oleg A. Ivanov and R.G. Burns.

Pi – Day – March – 14 as Pi = 3.14....

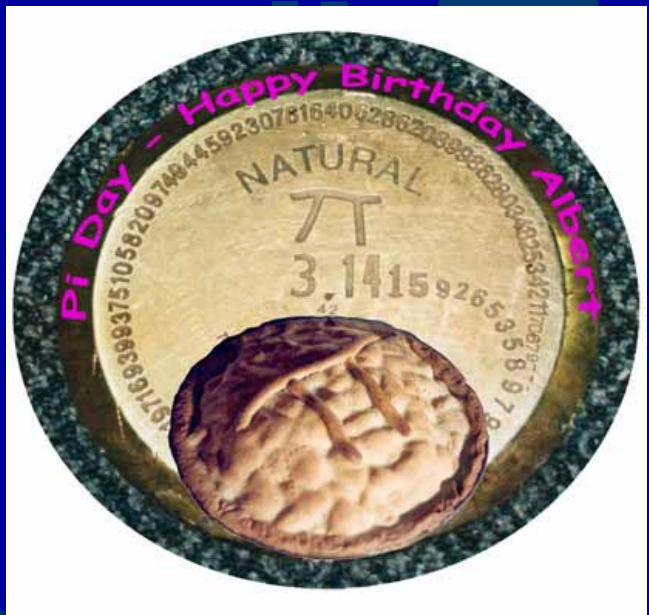


Larry Shaw, the founder of Pi Day at the Exploratorium - California

4000 – Pi clubs in America and Europe

Pi - π Day

Pi Day – celebrated every year in Europe and USA on March – 14 as π = 3.14.....



Pi - Shawl

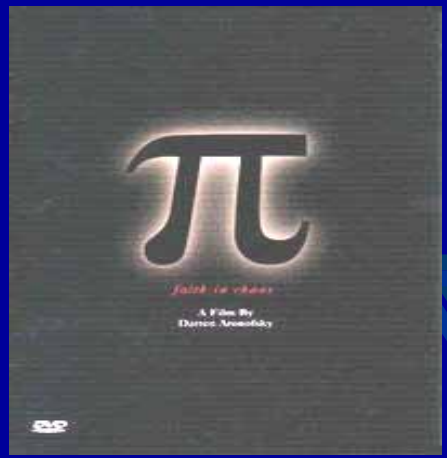
Name: Pi Shawl

Designer: Elizabeth Zimmerman

Pattern Source: Knitter's Best of Shawls and Scarves or Knitter's Almanac

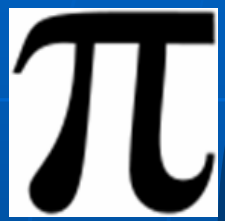
Yarn: lace weight Black Welsh Mountain Sheep wool

Hollywood Movie- π (1998)



Directed by	Darren Aronofsky
Produced by	Eric Watson
Written by	**Story:** Darren Aronofsky ,Sean Gullette ,Eric Watson **Screenplay:** Darren Aronofsky

Pi –Most Mysterious Puzzle



- God's riddle
- God's puzzle
- God's password
- God's secret
- God's Number
- God's symbol of our Perfect Spherical Expanding Universe

- Symbol of Our Perfect Spherical Universe

Mathematician Paul writes on Pi as:

"Pi is fundamental to the way in which our universe functions; practically *everything* *is dependent on Pi* at some basic level: light, sound, energy, gravity, electromagnetic fields, matter itself.....In fact pi is so central that it can be seen as a *symbol of our universe*.

Pi represents an omniscience which we will never possess, but that we can nudge closer and closer to as we approach its true value. Calculating pi as a rational value or exact or real value is a quest parallel to trying to fully understand our universe. It is for this reason that we wish to calculate pi to millions of places and beyond.

π - Unique Symbol of our Universe

π --- Unique Symbol of our Perefct Spherical Expanding Universe

What is accepted and believed today about Pi

1. **Pi is an irrational number.**
2. **Pi is a transcendental number and non-algebraic.**
3. **Pi is an approximate and estimated number.**
4. **Pi is an unending infinite decimal.**
5. **Is pi normal to base 10? We are not sure about it.**
6. **Circles cannot be squared – an unaccomplished dream of Greek mathematicians**
7. **Perfect circle does not exist**
8. **Perfect sphere does not exist**
9. **There is no definite pattern of numbers in pi.**
10. **Pi cannot be solved or calculated so we have to go with the approximations.**

Hundreds of approximations have been done so far to calculate the good approximate value of Pi

As a Civil Engineer, I know mathematically that:

Buildings build on wrong foundations are doomed to fall